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Analysis of Wave Propagation in Cold Plasma Medium

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Abstract: *Due to extremely wide range of plasma physics, able to classify the different physical behaviors, it becomes imperative to study the nature of wave propagation in different plasma medium. In recent years, however, researchers have developed techniques for producing low-temperature plasmas but a good low-temperature plasma source must be able to work at room temperature and atmospheric pressure. Therefore, in the present paper attempt has been confined to analyze the characteristics of wave propagation in cold plasma medium. For this purpose a dispersion relation is utilized, which relates the wave number 'k' to the wave frequency 'ω' and give all the information about the wave propagation. The theory of wave propagation in cold plasma medium referred to as the hydro magnetic extension of magneto ionic theory which is able to solve the problem of wave propagation in earth ionosphere.*

Keywords: *Magnet Ionic, Cold Plasma, Dispersion Relation, RCP and LCP.*

1. INTRODUCTION

Significant advances in the application of plasma in different fields such as engineering and communication, aerospace and satellite communication, biomedical science and to solve the problem of energy crises has incited with research world to establish the plasma physics as a separate entity in the field of research and development. At atmospheric pressure, most plasmas are so hot (thousands of degrees centigrade) that they would immediately kill any living cells they come into contact with. Moreover, these high-temperature plasmas are also very difficult to control. A important characteristics of plasma having ability to sustain a great variety of wave phenomena which are able to provide the significant information about the wave propagation. [1-2]. This information's are very useful for plasma diagnosis. This development of cold plasma can have far reaching effects not only in biomedical sciences, but also in all areas where plasma are used. Its portability has added benefit and help the scientists and researchers use plasma in other fields. Therefore in the present paper attempt has been motivated to characterize the wave propagation in specific cold plasma medium. The details of entire investigations are given in different section of this paper.

2. PROBLEM FORMULATION

To characterize the wave propagation in cold plasma medium, the concept of non magnetized and magnetized plasma and dispersion relation are utilized. In the case of propagation of waves in non magnetized plasma, effect of collision is ignored where as in magnetic plasma the effect of collision is taken account. This is due to fact that the magnitude and direction of velocities of particles changes when it comes near the plasma medium. For the cold plasma medium, the phase velocity must be greater than particles r m s velocity i.e.

$$v_p \gg v_{rms} = \sqrt{\frac{3K_BT}{m}} \tag{1}$$

Where, K_B = Boltzmann constant, T = Absolute temperature and m = mass of electron.

2.1. Non Magnetized Plasma

In this plasma the high frequency of plasma medium is considered and ion response is neglected. Now let us consider the electron motion in the plane electric field of a plane wave is given as [3]

$$m\frac{dv}{dt} = -i\omega mv = -eE \Rightarrow J = n_e ev = \frac{\omega_p^2}{\omega^2} i\omega\epsilon_0\, E = \sigma E \tag{2}$$

Where, $\frac{\omega_p^2}{\omega^2} i\omega\epsilon_0$ (conductivity), ω_p = plasma frequency, ω = operational frequency and ε_0 = permittivity of free space.

Now from Maxwell's fields equation

$$\nabla\times H = J + \frac{\partial D}{\partial t}$$

$$\Rightarrow ik\times H = \frac{\omega_p^2}{\omega^2} i\omega\epsilon_0\, E - i\omega\epsilon_0\, E = -i\omega\left(1-\frac{\omega_p^2}{\omega^2}\right)\epsilon_0\, E \tag{3}$$

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Thus the medium looks dielectric with $\in = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\in_0$

Let , $\frac{\omega_p^2}{\omega^2} = X$ hence index of refraction $n = \sqrt{1 - X}$

Also from the Faraday's and Amperes law, the Maxwell's law reduces to

$$kE_x = \omega\mu_0 H_y \tag{4}$$

And

$$kH_y = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\in_0 E_x \Rightarrow \omega^2 = \omega_p^2 + c^2k^2 \tag{5}$$

There phase velocity (v_p) and group velocity (v_g) can be given as

$$v_p = \frac{\omega}{k} = \frac{c}{\sqrt{1-X}} \text{ and } v_g = \frac{d\omega}{dk} = c\sqrt{1-X} \tag{6}$$

Now the condition for EM wave and plasma oscillation can be seen from Figure 1.

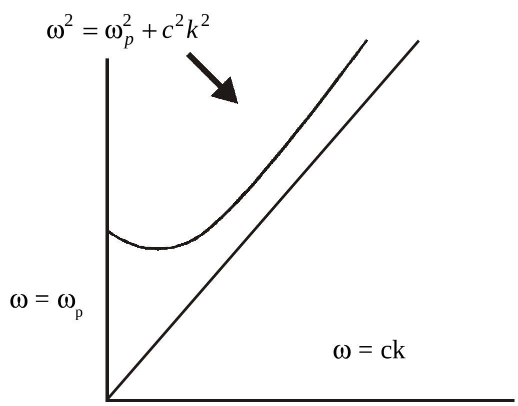

Figure 1: Wave Propagation in Non-magnetized Plasma

It is clear from Figure 1, if the plasma oscillation having the long wave length, the limit of wave propagation in the cold plasma medium. Hence for high frequencies the plane wave characteristics of a plasma degenerate to those of free space. This behavior is expected on a physical basis, because in the limiting area of infinite frequency electrons are unable to respond to the oscillating electron field.

2.2. Magnetized Plasma Medium

In the magnetized plasma medium the effect of collision is considered. For this purpose let us consider small perturbation i.e. B_1 ($B_1 \ll B_0$) and ions be movable. Now from the Maxwell's equations and Ohms law, the wave equation can be written as [4]

$$k \times (k \times E) = +\frac{\omega^2}{c^2} K.E = 0 \tag{7}$$

Where 'K' is dielectric tensor, since in plasma medium the propagated wave is circularly polarized in the left and right direction. Hence the dielectric tensor can be represented diagonally such as:

$$K = \begin{pmatrix} R & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & P \end{pmatrix} \tag{8}$$

Where R and L represents the permittivity for left and right circularly polarized waves respectively and P is identical to that of non magnetized plasma hence the actual dielectric permittivity is given as:

$$K = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \tag{9}$$

Where $S = (R + L) / 2$ and $D = (R - L) / 2$

Further consider the index of a reference can be represented by a vector (n = ck/ω). Thus the wave equation coordinate systems can be represented in tmensional as:

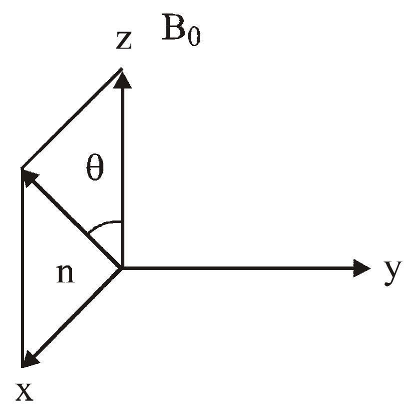

Figure 2:

$$k \times (k \times E) = +\frac{\omega^2}{c^2} K.E = 0 \Rightarrow$$

$$\begin{bmatrix} S - n^2\cos^2\theta & -iD & n^2\cos\theta\sin\theta \\ iD & S - n^2 & 0 \\ n^2\cos\theta\sin\theta & 0 & P - n^2\sin^2\theta \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \tag{10}$$

The non-trivial solutions of this wave equation is given as:

$$An^4 - Bn^2 + C = 0 \tag{11}$$

This is the dispersion equation in which

$$A = S\sin^2\theta + \cos^2\theta$$
$$B = RL\sin^2\theta + PS(1 + \cos^2\theta) \tag{12}$$
$$C = PRL$$

This is quite convenient formulation, as it allowed for propagation to different directions with respect to background magnetic field. Solving the above equation one has [5-6]

$$\tan^2\theta = \frac{-P(n^2-R)(n^2-L)}{(Sn^2-RL)\left(n^2-P\right)} \qquad (13)$$

For the special case of wave propagation parallel to the magnetic field (i.e., $\theta = 0^0$), the above equation reduces to

$$P = 0,\ n^2 = R \text{ or } n^2 = L \qquad (14)$$

Where as in the case of propagation perpendicular to the field (i.e., $\theta = 90^0$) equation yields

$$n^2 = RL/S \text{ and } n^2 = P \qquad (15)$$

3. PROBLEM SOLUTION

With the view of analysis of behavior of wave propagation in cold plasma the computational works were done using equations (12), (13), (14) and (15) respectively in non magnetized and magnetized field for RCP and LCP mode of propagation. The result thus obtained are plotted in the form of graphs Figures 2, 3 and 4.

4. CONCLUSION AND FUTURE WORK

From the analysis of wave propagation in cold plasma medium with the help of dispersion relation for non magnetized and magnetized field it is observed that there is significant effect of plasma medium on different mode of propagation. From equation (13) it may be concluded that $P^2 > 0$, it means that n is always real or purely imaginary. In this way cold plasma dispersion relation gives information about the wave propagation without evanescence. Due to two roots of opposite sign of refractive index (n) corresponding to a particular root for n^2, it is simply described that wave of same nature is propagating in the opposite direction.

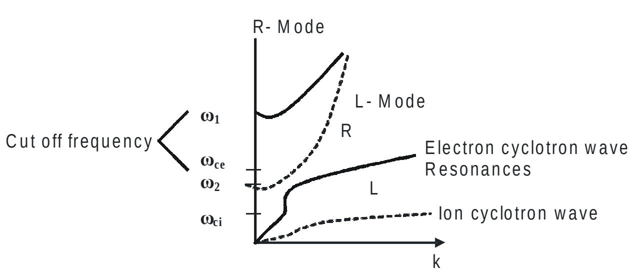

Figure 3: Dispersion Relation for RCP Mode of Propagation with $\omega_{pe} \ll \omega_{ce}$ Low Density Limit.

The dispersion curve for right hand wave propagating parallel to the equilibrium magnetic field in Figures 3 and 4, shows that the wave which propagate above the cut off frequency is a right hand circular polarized electromagnetic wave, which is modify by presence of cold plasma medium.

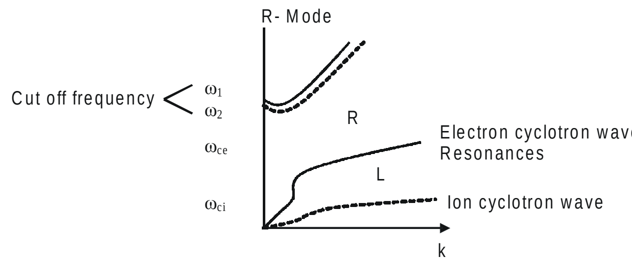

Figure 4: Dispersion Relation for RCP Mode of Propagation with $\omega_{pe} \gg \omega_{ce}$ High Density Limit.

It is also found that low frequency branch of dispersion curve differs fundamentally from the high frequency. This is due to fact that low frequency wave propagate through the plasma only in the presence of an equilibrium magnetic field, where as the higher frequency branch correspond to a wave can propagate in the absence of an equilibrium field.

From Figure 5, it is noticed that LCP wave propagating along the magneto static field (k ll B_0). This mode of propagation is affected by the presence of magnetic field. Further it may be concluded that lower hybrid reason is particularly important because these wave have resonance of both electron and ions, which provides efficient means of energy transfer between particle population.

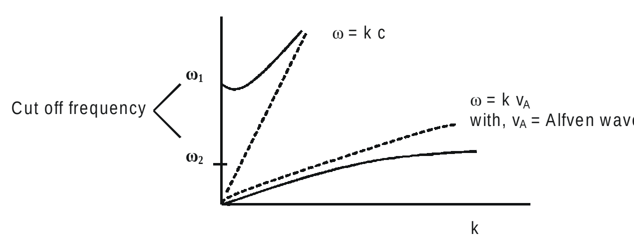

Figure 5: Dispersion Relation for a LCP Wave Propagating Parallel to the Magnetic Field in Magnetized Plasma.

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